Teaching Multiplication to a High-Functioning Autistic Child:

My Experience as an Educational Therapist

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Abstract

Very little has been written or published on how to teach mathematics to children with autism spectrum disorder. This is a short article about how, as an educational therapist, I successfully taught multiplication to a child diagnosed with high-functioning autism (HFA), despite some resistance from him. Using dice, cards, a timer, and water-soluble black and red ink markers to work out simple calculations, the child was taught to execute multiplication of numbers with two or more digits.

Introduction

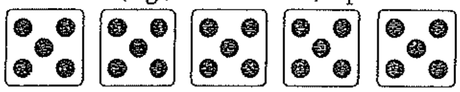
Children with autism tend to be keen on counting at an early age even before speaking other words. However, the first milestone in mathematical development is the emergence of one-to-one correspondence (i.e., counting objects one at a time and knowing that one number corresponds to one quantity). Such a physically based notion is how autistic children acquire mathematical concepts on a visual level (Siegel, 1996).

Just a year ago, one such child came to see me at a private learning clinic as his parents wanted me to prepare him for admission to a mainstream primary school, where there is a special needs officer working with such children. The child, already eight years old, was delayed a year entering school on the advice of his psychologist. Let me call the child Alan. He was diagnosed with HFA at the age of five. Alan would just sit in a corner for hours performing rapid addition and subtraction. Since no one knew how to communicate with him, he was not taught multiplication or division.

Fortunately, Alan had speech skills despite being diagnosed by a speech-language therapist as "very delayed." He could speak in English but no one (other than his parents) really understood what he was saying. He communicated better using the PECS (Picture Exchange Communication System) cards. While working with Alan, I learned he knew only addition and subtraction. The causes for confusion for many children, whether they are autistic or not, is learning arithmetic operations involves language to describe the operation (e.g., *add, plus, sum, total* are different words referring to the same symbol +). This difficulaty with language is where Alan was having a problem.

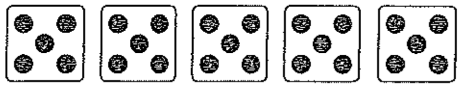
Dice and Number Flash Cards

One day, I noticed Alan enjoyed playing with big wooden dice left in a basket near my table. He would pick them up one at a time, placing them on his table in a row, with the same face (e.g., 5 black dots) upward as follows:

Dice: 

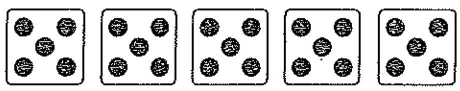
Then an idea struck me. I could try teaching Alan multiplication with dice. I picked up the number cards and placed them one at a time above each dice, as shown below.

Cards: 1 2 3 4 5

Dice: 

At first, Alan did not care what I was doing. After placing down the fifth card, I said, "There are five dice altogether." Then I started counting aloud the dots on each dice and wrote **5** with a water-soluble black ink marker on the table-top below each dice. Now, Alan began to take an interest in what I was doing and he repeatedly said "five" as I pointed to each dice. At that point in time, I wondered if Alan really knew what I was doing.

Cards: 1 2 3 4 5

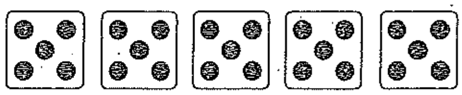
Dice: 

Ink: 5 5 5 5 5

"Good! Five dots on each dice," I said. "Now, how many dots are there altogether?"

Using a water-soluble red ink marker, I added + between the **5**s and put = at the end of the fifth **5**.

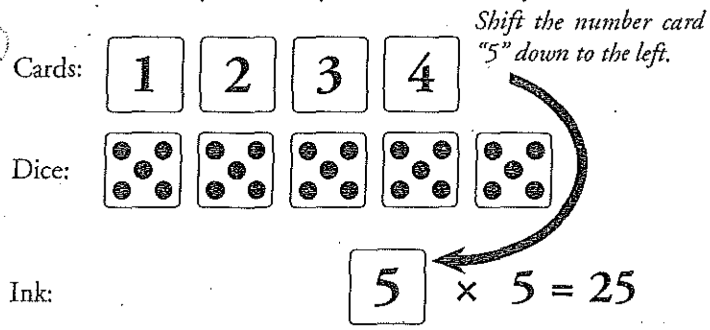
Cards: 1 2 3 4 5

Dice: 

Ink: 5 + 5 + 5 + 5 + 5 = 25

Alan counted aloud, "Five plus five plus five plus five plus five equal twenty-five." He took the black ink marker from me and wrote **25** at the end of the addition sum, without hesitating to think or check if his answer was correct. I was amazed. "Alan must be a genius," I thought. "Perhaps an autistic savant!"

I erased all except the fifth ink-written **5**, the = sign, and the answer **25**. Then I pointed at the last number card **5** and shifted it down next to the left of the ink-written **5**. I wrote × in water-soluble red ink and said, "Five dice (*pointing to the dice, one at a time*) of five dots (*pointing to the five dots, one at a time, on one dice*) each equal to five times (*pointing to the number card 5*) of five dots (*pointing to the ink-written 5 below the fifth dice*). And so five times (*pointing to the number card 5 again*) of five dots (*pointing to the ink-written 5 below the fifth dice again*) equals twenty-five (*pointing to Alan's ink-written 25*). That's how you can say it in another way."



"Twenty-five dots," said Alan nonchalantly, and he pointed to the answer, repeating exactly what I had said like a tape recorder.

Not knowing if Alan had really understood me, I repeated the activity with a different number of dice and each time with a different number of dots. Alan was able to give me correct answers. The boy was simply too good with his addition, and multiplication is really nothing more than a form of repeated addition, something many of us do naturally. Most of us are adding all the time in our minds. When I say "four times six," I am really just taking a short-cut to arrive at the same answer as six plus six plus six plus six.

As with all children, when teaching multiplication to children with autism, they need to understand what they are doing before moving on to the next level. However, one might ask, "What's the point?"

Although, Alan could easily perform rote multiplication at the level required for admission to a mainstream primary school. However, if he could not use the operation functionally, little had been accomplished except a demonstration of his ability to memorize, thereby impressing the people around him. It is important, as Siegel (1996) pointed out, "to keep comprehension commensurate with performance when learning mathematics" (p. 282).

Moving Beyond Dice and Number Flash Cards

The next task was to move Alan beyond doing multiplication with dice and number cards. I taught him 0×, 1×, 2×, 5×, and 10× tables the usual way I would with nonautistic children. However, for reasons unknown, Alan seemed to be somewhat resistant to learning 3×, 4×, 6×, 7×, 8×, 11×, and to 12× tables. I began to think of another way to get him to learn or memorize these other tables. For example, the 7× table can be a sum of 2× and 5× tables, i.e., 7× = 2× + 5×. I shall elaborate on this later on in examples.

"That's all he needs to know," I told his parents one day, after working three sessions with Alan. "These times tables allow him to perform multiplication with any numbers."

Alan managed to acquire the 0×, 1×, 2×, 5×, and 10× tables by rote. This is often observed in autistic children, who seem to store lists of items in memory for prolonged periods in the exact form in which they were first experienced, without changing them in any way (Baron-Cohen & Bolton, 1993).

One possible explanation was that Alan might have a unique way of understanding how the multiplication system works. This can be explained by the child's superior systemizing ability, which has been described by Baron-Cohen and Wheelwright (2004) as "the drive to analyze and build systems in order to understand and predict the behavior of impersonal events or inanimate or abstract entities" (p. 18). Systems are all around us and there are at least six kinds: mechanical systems (e.g., machines), natural systems (e.g., biological phenomena), abstract systems (e.g., mathematics), motoric systems (e.g., piano finger technique), organizable systems (e.g., library catalogue), and social systems (e.g., business). In Alan's case, he displayed superior facility with abstract systems and was able to see patterns in times tables that he had been taught. He knew that 0 times whatever number is always a zero. To him, the 2× table sounded like an endless chant (and Alan would go on chanting unless he was told to stop). 5× table thrilled Alan with 5 and 0 repeating themselves down the table; he could tell me that an odd multiplicand results in a product with a 5 in its unit (e.g., 7×5 = 35), but an even multiplicand has a product ending with a "0" in its unit (e.g., 8×5 = 40). For 10× table, Alan learned to add a 0 to the right end of a multiplicand.

By the seventh session, Alan was doing multiplication with repeated addition to obtain his answers. For instance, 12 ×15 = ?, where 12 is a multiplicand, 15 is a multiplier, and ? is the unknown product to be calculated.

The multiplier 15 can consist of the following options:

(a) 10 + 5 *or*

(b) 10 + 2 + 2 + 1 *or*

(c) 5 + 5 + 2 + 2 + 1

... based on the 1×, 2×, 5×, and 10× tables that Alan had already known. Let's say that Alan decided to choose the option (b): 10 + 2 + 2 + 1. He wrote down four separate alternative multipliers: one column under the 10× table, two columns under the 2× table, and one column under the 1× table. Then he worked out his answers, beginning with the multiplicand 1 and moved numerically and sequentially downward until he arrived at the multiplicand 12 to obtain his answer (see below).

Option (b)

Cardinals		Multiplier		Time-tables		Product
				10×2×2×1×		
1	×	15	=	10+2+2+1		
2	×	15	=	20+4+4+2		
3	×	15	=	30+6+6+3		
4	×	15	=	40+8+8+4		
5	×	15	=	50+10+10+5		
6	×	15	=	60+12+12+6		
7	×	15	=	70+14+14+7		
8	×	15	=	80+16+16+8		
9	×	15	=	90+18+18+9		
10	×	15	=	100+20+20+10		
11	×	15	=	110+22+22+11		
12	×	**15**	=	**120+24+24+12**	=	**180**

By the nineteenth session, Alan was learning to compute his answers using a short-cut method. This was achieved after much resistance from the child, who still preferred the long, ritualistic way of doing repeated addition. However, for practical reasons (e.g., during a class test that has to be completed within a specified time), I saw the need for Alan to work out his answers without having to go through a long series of repeated addition. Explaining to Alan why he had to take the shortest route to do his multiplication was tough. The child neither understood why he needed to do that nor knew what I meant. It took me another three sessions before I came up with the idea of using a timer.

Alan had to work against the timer to see how much time he would need to complete the multiplication. He often enjoyed "playing" with the timer. (Although I have to admit that there were unpleasant times when Alan became frustrated for one reason or another, threw temper tantrums, and refused to work with the timer). But usually I was able to convince him to do multiplication with the short-cut method. For instance, using the same 12×15 = ?, Alan learned to directly multiply the multiplicand 12 using the smaller alternative multipliers, instead of the bigger multiplier 15, through one of the three possible routes:

(a) 10 and 5 *or*

(b) 10, 2, 2, and 1 *or*

(c) 5, 5, 2, 2, and 1.

Say, Alan chose the second route and mentally computed his answer with the familiar 10×, 2×, and 1× tables.

Route (b)

12 × 15 = **?**

where 15 = 10 + 2 + 2 + 1

× ↓ ↓ ↓ ↓ where 12 ×10 =120

× where 12 ×2 =24

× where 12 ×2 =24

and 120 + 24 + 24 + 12 = 180

By the year's end, Alan was able to perform multiplication with numbers of three or more digits easily without a hitch. An impressive feat indeed!*

* Note: Since no two autistic children will be alike, we cannot expect every one of them to learn or be taught in the same way that Alan was. However, the approach can be modified or adapted to meet individual learning needs.

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